Midterm Exam (100 points)

1. Concepts (16 points total):

Define the following concepts in a few short sentences (**4 pts each**):

a) valid argument (give the telling definition that allows us to see why tautologies follow from everything and why from a contradiction, everything follows)
b) sound argument
c) tautological consequence
d) truth-functional connective

2. True, false, or meaningless (24 points total):

Indicate whether the following sentences are true or false and ***explain*** your answer in one or two sentences. If something is wrong with the sentence, explain what it is. (**3pts each**)

a) Every logical truth is a tautology.
b) "Home(max) $\vee$ Home(claire)" is a proper sentence of FOL (i.e. of its pet language dialect).
c) Every sound argument is true.
d) In FOL, every object in the domain of discourse must be associated with precisely one individual constant.
e) "$\neg$ A $\vee$ B" is logically equivalent to "A $\rightarrow\neg\neg$B"
f) Under no assignment of truth values to "A" can the sentence "$\neg$ (A $\rightarrow$ $\neg$ A)" be false.
g) If you conjoin a given claim with that very claim that has been negated 30549 times, the resulting conjunction is a contradiction.
h) A system of logical inference rules is sound just in case it allows one to derive, from any given set of premises, each of that set's tautological consequences.

3. **Translations (16pts total)**:

(i) <u>Translate sentences (a) – (e) from English into FOL, using the following names and predicates (</u>**2pts each**).

b: the butler **g**: the gardener **l**: the logic instructor
p: the perpetrator **v**: the victim
Poi(x): x was poisoned **Str**(x): x was strangled to death
Ins(x): the perpetrator drove x insane and x died as a consequence

(a) The perpetrator is the butler if the victim was poisoned.
(b) It is either both true that the victim is the gardener and the butler is the perpetrator, or the logic instructor is the victim. **[inclusive or]**
(c) The victim is not the perpetrator (hence: it was not a suicide).
(d) Neither the gardener nor the butler were strangled.

(ii) <u>Translate the following sentences of FOL into natural-sounding English, using the interpretation of the names and predicates listed in 3 (i) (</u>**2pts each**<u>):</u>

(e) $[\text{Poi}(v) \lor \text{Str}(v) \lor \text{Ins}(v)] \land \neg [\{\text{Poi}(v) \land \text{Str}(v) \land \text{Ins}(v)\} \lor \{\text{Poi}(v) \land \text{Str}(v)\} \lor \{\text{Poi}(v) \land \text{Ins}(v)\} \lor \{\text{Str}(v) \land \text{Ins}(v)\}]$ **[Note: much easier than it looks]**
(f) $[p = b \lor p = g \lor p = l] \land [p = b \leftrightarrow (p \neq g \land p \neq l)] \land [p = g \leftrightarrow (p \neq b \land p \neq l)] \land [p = l \leftrightarrow (p \neq b \land p \neq g)]$ **[Try to find a crisp English translation.]**
(g) $\text{Ins}(v) \rightarrow v = l$ **[use “only if,”** you only get 2pts if you do, max 1pt otherwise**]**
(h) $p \neq g \rightarrow \neg \text{Str}(v)$ **[use „unless,“** max 1pt if you don’t**]**

4. **Proof challenge 1: exonerate the logic instructor (10 pts total)**

A friend of yours serves as a detective in the Pittsburgh police force. Seeking your help, he informs you of a case that involves the following three individuals: the butler, the gardener, and the logic instructor. As he presents you with some of the facts he has discovered so far, your logic skills kick in. At some point, you cannot but exclaim: "Ha, so the logic instructor is innocent, then!" Your friend has not finished citing all the relevant facts yet. And though he smiles knowingly, he is nevertheless impressed by your deductive skills. He asks: "That is true, but how did you know?"

Prove the logic instructor's innocence, i.e. **prove: "$p \neq l$"** using only

3 a, 3c, 3f-h, and 3e*: "$Poi(v) \vee Str(v) \vee Ins(v)$."

Give a ***formal proof*** using the Fitch representation style. As you do, make sure to number the lines and justify each step in the proof by citing the appropriate rule used and the line numbers.

Hint: Your main strategy here is a reductio. Within that strategy, you begin by isolating some relevant material from the premises that you can then use to derive a number of useful claims. You then pursue a proof by cases to derive the desired contradiction.

Note: You need correct translations of 3 (i) a) and c) to succeed. If you are uncertain about your translation, you can get the correct translations from me (of course, you then lose up to 2x2 pts for the translation task).

5. **Revealing truth tables (12pts total)**

As it turns out, the logic instructor is not just innocent; he is the victim (which explains your friend's previous smile). Moreover, both the gardener and the butler are obsessive logicians and refuse to answer questions that don't stimulate their logical abilities. This, your friend confesses, makes it difficult for him to question the two. The following fact makes things worse: he has been told that the two (a weird side-effect resulting from having been exposed to too many logic puzzles) have developed a somewhat quirky characteristic: one of them always tells the truth, the other one is a pathological liar. However, so far, your friend hasn't been able to determine who's who.

Ready to help your friend out, you say: "No problem, just ask them to answer the following questions:

a) Are "$A \rightarrow \neg (A \wedge B)$" and "$(A \leftrightarrow \neg B) \vee \neg A$" tautologically equivalent?
b) Considering

(1) "$\neg B \vee C$," (2) "$B \rightarrow A$," and (3) "$\neg A \vee C$"

…is it both true that (1) is a tautological consequence of (2) and (3) and that (3) is a tautological consequence of (1) and (2)?

c) Is "$(\neg B \rightarrow (A \wedge B)) \vee \neg B$" a tautology?

The ***butler's*** responses are: a) no, b) yes, c) no.
Predictably, the ***gardener's*** responses are: a) yes, b) no, c) yes.

Construct truth tables to determine the correct answers to a-c and **explain** how the truth tables settle your response **(4pts each, 2pts TT, 2pts explanation).**
[You should now know who the truth teller is and who the liar.]

6. **Proof- challenge 2 (18 pts total)**:
The two suspects are thrilled that your friend, the detective, is giving them some logic tasks. However, they insist that he also show his ability to formally ***derive a few claims in Fitch***. Help him (**6pts each**):

- **a. Prove "**$\neg (A \vee B)$" from "$\neg A \wedge \neg B$"
- **b. Prove** (without premises) that **"**$\neg(A \wedge (B \rightarrow \neg A) \wedge \neg\neg B)$" is a tautology.
  [**hint**: this, recall, requires using a reductio strategy]
- **c. Prove** "$A \leftrightarrow (\neg B \wedge \neg C)$" from "$B \rightarrow C$" and "$\neg A \leftrightarrow C$"
  [**hint**: you start by assuming A and then use "$\neg A \leftrightarrow C$" to get "$\neg C$" in a few steps… use that with "$B \rightarrow C$" to get "$\neg B$" and, thus, "$\neg B \wedge \neg C$" … if you manage to do this, proving the other direction should be easy]

7. **The murder weapon (4pts total)**

   Satisfied with your responses, the gardener confesses that he has indeed purposefully driven the logic instructor insane (the butler, of course, denies that the gardener did it and claims the murder for himself). The motif? The gardener wanted to be logic instructor instead of the logic instructor. He thought that the logic instructor, who firmly – indeed, obsessively – believed that every claim is either true or false, was unfit to do his job and help his students move on to the interesting study of many-valued logic. To push the logic instructor over the edge, the gardener sent him the following anonymous note:

   "A crocodile has stolen a child. It promises the mother that her child will be returned ***if and only if*** *she correctly predicts what the crocodile will do next. "You will not return my child," the mother replies. The crocodile, honor-bound by its promise, what will it do?"*

   Given what you know about the logic instructor, ***explain to your friend why this note drove the logic instructor insane.***

Bonus I (4pts total):

Using both letters A and B (to symbolize atomic sentences), generate

A disjunction that disjoins a) a biconditional connecting a negated and a non-negated atomic sentence and b) a conjunction of two conditionals in such a way that the resulting disjunction is TT-possible, yet not a tautology. [***Don't overthink, just try***.]

Bonus II (max. 6pts / 0.5 pts per ?)

Complete the following proof by filling in the rules, line numbers, and the required steps as appropriate. (Copy to your blue book.)

1.	$A \vee (B \rightarrow (C \rightarrow A)$	
2.	$\neg A$	-
3.	**?**	-
4.	**?**	**?**
5.	$B \rightarrow \neg C$	**?**
6.	$(B \rightarrow (C \rightarrow A)$	-
7.	B	-
8.	**?**	$\rightarrow$Elim: 6, 7
9.	**?**	-
10.	A	**?**
11.	$\perp$	**?**
12.	$\neg C$	**?**
13.	$B \rightarrow \neg C$	$\rightarrow$Intro: 7-12
14.	**?**	**?**
15.	$\neg A \rightarrow (B \rightarrow \neg C)$	**?**